

CENTRO DI RICERCHE E DOCUMENTAZIONE  
“PIERO SRAFFA”

UNIVERSITÀ DEGLI STUDI ROMA TRE

THE MARKET FOR SAVINGS  
IN THE THEORY OF GENERAL  
INTERTEMPORAL EQUILIBRIUM

Sergio Parrinello

Quaderno di Ricerca n. 8  
2009



### *Abstract of the paper*

*A recent debate on the theory of general intertemporal equilibrium with production is focused on whether this theory is immune from the criticism to the aggregate version of the neoclassical theory of value and distribution. This article resumes two controversial and related issues of that debate: 1) whether a market of aggregate values (saving) for each period is implicit in that theory and is as much relevant for the determination of an equilibrium as the markets for dated physical commodities which appear in the generally accepted form of the corresponding model; 2) whether the possibility of reverse capital deepening and reswitching of techniques can intrude into the model through that hidden market and become a source of non meaningful equilibria. The arguments presented will lead to an affirmative response to question 1). Furthermore they will provide, also in the light of a recent (2009) contribution by Garegnani to the same question, a revised version of the quasi-equilibrium model which he used to describe the possibility 2).*

# THE MARKET FOR SAVINGS IN THE THEORY OF GENERAL INTERTEMPORAL EQUILIBRIUM

Sergio Parrinello

## Introduction\*

The recent debate on the theory of general intertemporal equilibrium among Garegnani (2000; 2003), Mandler (2005), Parrinello (2005; 2008), Foley (2008), Petri (2004) and Schefold (2008) seems to be centred on two main issues.

- 1) It has been questioned whether a value aggregation of physical quantities is necessary in the theory of general intertemporal equilibrium; in particular whether an equilibrium condition between aggregate saving and aggregate investment for each period is as much determinant as an equilibrium condition in the market of a physical good or service.
- 2) It has been debated whether the possibility of *reverse capital deepening* and *reswitching* of techniques, which was proved for the aggregate version of the neoclassical theory of capital and distribution, can be a *specific* source of non meaningful intertemporal equilibria or, instead, can be neutralized by the same sufficient conditions (e.g. the weak axiom of revealed preferences or the representative consumer) that since long time ago have been adopted to prove the existence, uniqueness and stability of general equilibrium.

The debate on issues 1), 2) will be resumed in the light of Garegnani (2009). We shall use the abbreviation “Intertemporal” instead of “model of general intertemporal economic equilibrium”; and the abbreviation “*A*-temporal” instead of “model of general a-temporal economic equilibrium”. The adjective “a-temporal” encompasses the terms “static” or “one-period”, although we are aware of possible objections to the use of such expressions interchangeably. Clearly question 2) above arises only if the answer to the former 1) is that aggregation is necessary. Ultimately the controversy seems to resolve itself into the acceptance or refusal of a syllogism of the following type. 1) Each *In-*

\* This article has benefited from live discussion with Garegnani and Schefold and from mail exchanges with Foley and Petri on its earlier drafts. Furthermore the author acknowledges the suggestions received from two anonymous referees. The responsibility for errors and omissions remains only mine.

tertemporal can be formally converted into an  $A$ -temporal and shares the same properties, in terms of existence, uniqueness and stability of equilibrium which have been already proved for the static general equilibrium model. 2) Hahn–Garegnani’s model, which is taken as a prototype for the discussion at issue, is an *Intertemporal*. 3) Hence Hahn–Garegnani’s model is formally equivalent to an  $A$ -temporal and cannot exhibit properties which require the demonstration of special theorems.

The premise 1) of the syllogism above is ambiguous because it presupposes that all  $A$ -temporals have a unique analytical structure and the same equilibrium properties. Schefold (2008) has convincingly demonstrated how an *Intertemporal* can be reduced to an  $A$ -temporal and that any solution to the former must be also a solution to the latter. However, the second part of the premise 1) does not follow from the first one. The one-period or static model to which an *Intertemporal* is converted possesses a special structure which reflects that of its ancestor. The existing theorems of existence, uniqueness and stability, which have been proved for a standard  $A$ -temporal without capital used for production, may not be extendable to the  $A$ -temporal corresponding to the *Intertemporal*. Garegnani (2000; 2003) has argued that non meaningful equilibria can exist in the *Intertemporal* as a result of *reverse capital deepening* and *reswitching* of techniques. Elsewhere, I (Parrinello 2005; 2008) stressed that the theorems of *tâtonnement* stability cannot be extended from the  $A$ -temporal to the *Intertemporal*, because the adjustment mechanism is different in the two models. We should specify more carefully what is the structure of the *Intertemporal*, or its equivalent  $A$ -temporal, which opens the door to different equilibrium and disequilibrium properties related to the theory of capital.

Sections 1, 2, 3 summarize the elementary notions of one period versus multi-period budget constraints, of dependence among equilibrium conditions and of perfect substitutes — as an introduction to the main argument. We shall reiterate in sections 4, 5 that a market for saving may and in a sense *must* exist in each period of the *Intertemporal* and intervenes in a special way in the determination of equilibrium. Sections 6a, 6b specify the individual behaviour which underlies the markets for saving in the *Intertemporal*. We shall deal in section 7 with the distinction among dated Walras laws (see Garegnani, 2009). Sections 1, 2, 3, 4, 5, 6, 7 should convince the reader that a market for saving must exist in each period of the *Intertemporal* if its structural form has to be reduced into an  $a$ -temporal form in which each agent is subjected to a unique budget constraint. More importantly, the properties of the market demand and supply functions of the corresponding  $A$ -temporal cannot be assumed as if the saving markets would not exist, but they must be derived from the properties of the demand and supply functions of the *Intertemporal* taken in its structural form. Section 8 clarifies the role