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Abstract

In this paper we consider the optimal design of induction electric motors which can be formulated as a mixed variable programming problem. Two different solution strategies have been used to solve this problem and the obtained results have been analyzed and compared. They are very interesting and show the fruitfulness of directly taking into account the presence of both continuous and discrete variables.

1 Introduction

Three-phase induction motors are widely used in industrial applications, and have a significant impact on electricity consumption.

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The European Committee of Manufacturers of Electrical Machines and Power Electronics and the European Commission have agreed to a joint classification scheme that will enable the customers and users of induction motors to have a simple appreciation of the efficiency of these components. The classification scheme foresees three efficiency classes (eff1, eff2 and eff3): the lowest class is the "standard efficient", the middle could be defined "energy efficient" while the highest is the "high (premium) efficient".

This classification needs to develop new ranges of motors. The design of "high efficiency" induction motors requires the use of specific optimization techniques. Indeed, the physical and mathematical principles which are at the basis of the design of induction motors, are insufficient to produce a near optimal design. Moreover, with the increasing cost of electrical energy and concurrent development in the material technology, the operating cost and/or some specific items of performance play a significant role in the overall economics as well as in the efficiency of the system.

The growing demand of high-performance motors requires the definition of more and more efficient designs. The only possibility of obtaining such kinds of motors is to use automatic optimization procedures along with the definition of an analytical model of the motor itself. Such a model can be obtained by reducing the physical description of the motor to equivalent parameters such as resistances and inductances [2]. The adopted analytical model takes into account the influence of saturation on stator and rotor reactances, the influence of skin effect on rotor parameters and the effects of temperature on motor resistances.

In Section 2 we describe the optimal design problems which arise in the design of induction motors. Such problems can be naturally stated as mixed-variable programming problems. Section 3 is devoted to a continuous approach used for solving the problem. In Section 4 we introduce the mixed-variable programming algorithm which has been used to tackle the optimal design problem. Finally, in Section 5 we describe the results obtained by using the mixed-variable approach.

In the paper we denote by $|\cdot|$ the cardinality of a set.
2 Problem description

The optimal design of electric motors requires particular attention in the choice of the objective function that usually concerns economic or performance features. In order to cover both these aspects of the design problem, we have chosen two objective functions that can affect the design optimization of three-phase induction motors. Particularly:

\[ f^{(1)} : \text{Manufacturing cost (to be minimized)}; \]
\[ -f^{(2)} : \text{Rated efficiency (to be maximized)}. \]

The induction motor is completely determined by the following independent variables which define the stator and rotor dimensions. They are:

- the stator slot height \((x_1)\)
- the stator tooth width \((x_2)\)
- the rotor slot height \((x_3)\)
- the rotor tooth width \((x_4)\)
- the air-gap length \((x_5)\)
- the air-gap flux density \((x_6)\)
- the inner stator diameter, \((y_1)\)
- the stack length \((y_2)\)
- the outer stator diameter \((y_3)\)
- the stator wire size \((y_4)\)
- the electrical steel type \((t)\)

Our aim is to design a motor without affecting heavily the tooling costs and the building process. For this reason some of the above quantities should assume a finite number of values. This is essentially due to the fact that the preexistent lamination punch tools or stator housing tools allow to handle only some prefixed values of the independent variables. Obviously, if we want to change all motor dimensions and renew the lamination tooling, then these kind of limitations can be neglected. In the former case however, the corresponding variables must be considered as discrete variables.
whose feasible values are related to the base components availability and to the limitations of the existing manufacturing process. As for the continuous variables, their values must be within given bounds which are connected to mechanical and technological constraints, according to the manufacturer suggestions.

The variable $t$ represents the electrical steel type that plays a significant role on the motor performance: its right choice, combined with the design optimization of the motor, should allow to achieve better results and higher efficiency. The choice of a “suitable” electrical steel type depends on several aspects such as cost, workability, “business tradition” and storehouse demands. In this study six “fully processed” commercial steels have been considered, labelled with $0, 1, \ldots, 5$ (where $t = 0$ represents an “high performance” and “high cost” steel type while $t = 5$ represents a “low performance”, “low cost” steel type).

Beside the bound constraints on the variables, the problem involves also some nonlinear constraints which concern mainly the motor performances. In particular, they are: the stator wind-
ing temperature, the rotor bars temperature, the flux density in the stator and rotor teeth, the rated slip, the starting torque, the starting current, the breakdown torque, the power factor at rated load and the stator slot fullness.

Finally, depending on the choice of the objective function \( f(l), l = 1, 2 \), we come up with the following mixed variable programming problems for a 7.5 kW, 4 pole, 380 V, 50 Hz, three-phase induction motor,

\[
\begin{align*}
\min_{x,y,t} & \quad f(l)(x, y, t) \\
g(x, y, t) & \leq 0 \\
16.0 & \leq x_1 \leq 19.0 \\
4.5 & \leq x_2 \leq 6.5 \\
16.0 & \leq x_3 \leq 18.5 \\
3.5 & \leq x_4 \leq 5.0 \\
0.3 & \leq x_5 \leq 0.5 \\
0.5 & \leq x_6 \leq 0.68 \\
y_1 & \in \{126.6, 131.6\} \\
y_2 & \in \{140, 150, 160, 170, 180, 190\} \\
y_3 & \in \{180, 200, 220\} \\
y_4 & \in \{1.4, 1.45, 1.5, 1.55, 1.5727, 1.6, 1.65, 1.7, 1.75\} \\
t & \in \{0, 1, 2, 3, 4, 5\},
\end{align*}
\]

where \( x = (x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6)^T \) and \( y = (y_1 \ y_2 \ y_3 \ y_4)^T \).

The distinguishing features of these problems are reported below.

(i) To evaluate the objective and constraint functions on a given point, it is necessary to perform a numerical simulation of the motor operating status. For this reason, an explicit representation of the objective and constraint functions is not available.

(ii) The constraints \( g(x, y, t) \leq 0 \) are not very restrictive, namely, it is relatively easy to find a feasible point and to remain in the feasible region.

(iii) \( y \) and \( t \) can assume only a finite number of values. In particular, the discrete variable \( t \) affects the structure of the objec-
tive and constraint functions. Moreover, it cannot assume any intermediate value since for such values the corresponding optimization problem is undefined.

Taking into account property (ii), we can transform Problem \((P^l)\) using a standard technique to eliminate the nonlinear constraints (see, for instance, [5]). In particular, we use these new objective functions.

\[
\tilde{f}^{(l)}(x, y, t) = \begin{cases} 
    f^{(l)}(x, y, t) & \text{if } g(x, y, t) \leq 0 \\
    +\infty & \text{otherwise}
\end{cases} \quad l = 1, 2.
\]

Hence we consider problems

\[
\min_{x,y,t} \tilde{f}^{(l)}(x, y, t) \\
\quad l_x \leq x \leq u_x \\
\quad y_i \in Y_i, \quad i = 1, 2, 3, 4 \\
\quad t \in T,
\]

\((\tilde{P}^l)\)

for \(l = 1, 2\), where

\[
l_x = \begin{pmatrix} 
    16.0 \\
    4.5 \\
    16.0 \\
    3.5 \\
    0.3 \\
    0.5 
\end{pmatrix} \quad u_x = \begin{pmatrix} 
    19.0 \\
    6.5 \\
    18.5 \\
    5.0 \\
    0.5 \\
    0.68 
\end{pmatrix}
\]

and

\[
Y_1 = \{126.6, 131.6\} \\
Y_2 = \{140, 150, 160, 170, 180, 190\} \\
Y_3 = \{180, 200, 220\} \\
Y_4 = \{1.4, 1.45, 1.5, 1.55, 1.5727, 1.6, 1.65, 1.7, 1.75\} \\
T = \{0, 1, 2, 3, 4, 5\}.
\]
3 Continuous approach

A first attempt consists in solving the following nonlinear continuous optimization problems

\[
\min_{x,y} \tilde{f}(l)(x,y,t)|_{t=h} \\
l_x \leq x \leq u_x \\
l_y \leq y \leq u_y,
\]

for \( l = 1, 2 \), obtained by setting \( t = h \), for all \( h = 0, 1, \ldots, 5 \) and relaxing, in a suitable way, the discrete variables \( y \) and where

\[
l_y = \begin{pmatrix} 126.6 \\ 140.0 \\ 180.0 \\ 1.4 \end{pmatrix}, \quad u_y = \begin{pmatrix} 131.6 \\ 190.0 \\ 220.0 \\ 1.75 \end{pmatrix}.
\]

Problems \((\tilde{R}_h^l)\) for \( h = 0, 1, \ldots, 5 \) and \( l = 1, 2 \) can be rewritten as

\[
\min_z \varphi(z) \\
l_z \leq z \leq u_z,
\]

where \( z = (x,y) \in \mathbb{R}^n_z \), \( l_z = (l_x, l_y) \) and \( u_z = (u_x, u_y) \).

By property (i) of Problem \((P^h)\), an explicit representation of \( \varphi(z) \) is not available. Hence, to solve Problem (1) we applied the derivative free algorithm proposed in [4] whose description is reported below.

---

**Procedure DFA**

**Data.** \( \alpha^0 > 0 \).

1. Set \( j = 1 \).
2. Apply procedure \( DF(n_z, z^j, \alpha^{j-1}, z^{j+1}, \alpha^j) \).
3. If \( \alpha^j > \alpha_{\text{tol}} \) then set \( j := j + 1 \) and go to step 2. else return \( (z^{j+1}, \alpha^j) \).
We refer to [4] for the theoretical analysis of Procedure DFA.

Procedure DF\((n_z, z, \mu^0, \tilde{z}, \mu)\)

**Data.** \(\gamma > 0, \delta \in (0, 1), \delta_1 \in (0, 1), \theta \in (0, 1), d^i = e^i\) and \(\bar{\alpha}^i = \mu^0\) for \(i = 1, \ldots, n\).

1. **Initialization:** Set \(i = 1\) and \(z^i = z\).

2. **Direction choice:**
   
   2.1 Compute \(\alpha^i_{\text{max}}\) s.t. \(z^i + \alpha^i_{\text{max}}d^i = u_z^i\) and set \(\alpha = \min\{\bar{\alpha}^i, \alpha^i_{\text{max}}\}\).  
   
   If \(\alpha > 0\), \(\varphi(z^i + \alpha d^i) \leq \varphi(z^i) - \gamma(\alpha)^2\),  
   
   then go to Step 4.

   2.2 Compute \(\alpha^i_{\text{max}}\) s.t. \(z^i - \alpha^i_{\text{max}}d^i = l_z^i\) and set \(\alpha = \min\{\bar{\alpha}^i, \alpha^i_{\text{max}}\}\).  
   
   If \(\alpha > 0\), \(\varphi(z^i - \alpha d^i) \leq \varphi(z^i) - \gamma(\alpha)^2\),  
   
   then set \(d^i = -d^i\) and go to Step 4.

3. **Direction failure:** Set \(\bar{\alpha} = 0\), \(\bar{\alpha}^i = \theta\alpha\), and go to Step 5.

4. **Linesearch:**

   4.1 Let \(\hat{\alpha} = \min\{\alpha^i_{\text{max}}, \frac{\alpha}{\gamma}\}\).  
   
   If \(\alpha = \alpha^i_{\text{max}}\) or \(\varphi(z^i + \hat{\alpha} d^i) > \varphi(z^i) - \gamma \hat{\alpha}^2\),  
   
   then set \(\bar{\alpha} = \alpha\), \(\bar{\alpha}^i = \alpha\) and go to step 5.

   4.2 Set \(\alpha = \hat{\alpha}\) and go to step 4.1.

5. **New point:** Set \(z^{i+1} = z^i + \bar{\alpha} d^i\).

6. **Stopping criterion:** If \(i = n_z\), then \(\tilde{z} = z^{i+1}\), \(\mu = \max_{i=1,\ldots,n}\{\bar{\alpha}^i, \delta_1 \mu^0\}\)  
   
   else set \(i = i + 1\) and go to Step 2.

Every problem has been solved starting from the initial points