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G. Ausiello¹, M. Demange², L. Laura¹, and V. Paschos³

1 Dip. di Informatica e Sistemistica Università di Roma "La Sapienza" Via Salaria 113 00198 Roma Italy. {ausiello, laura}@dis.uniroma1.it
2 ESSEC demange@essec.fr
3 Université Paris-Dauphine, Place du Maréchal De Lattre de Tassigny, 75775 Paris Cedex 16, France paschos@lamsade.dauphine.fr

Abstract. The Quota Traveling Salesman Problem is a generalization of the well known Traveling Salesman Problem. The goal of the traveling salesman is, in this case, to reach a given quota of the sales, minimizing the amount of time. In this paper we address the on-line version of the problem, where requests are given over time. We present algorithms for various metric spaces, and analyze their performance in the usual framework of the competitive analysis. In particular we present a 2-competitive algorithm that matches the lower bound for general metric spaces. In the case of the half-line metric space, we show that it is helpful not to move at full speed, and this approach is also used to derive the best on-line polynomial time algorithm known so far for the more general On-Line TSP problem (in the homing version).

1 Introduction

Let us imagine that a traveling salesman is not forced to visit all cities in a single tour but in each city he can sell a certain amount of merchandise and his commitment is to reach a given quota of sales, by visiting a sufficient number of cities; then he is allowed to return back home. The problem to minimize the amount of time in which the traveling salesman fulfills his commitment is known as the Quota Traveling Salesman Problem (QTSP for short, see [4, 9] for a definition of the problem) and it is also called Quorum-Cast problem in [11]. Such problem can be seen as a special case of the Prize-Collecting Traveling Salesman Problem (PCTSP⁴, [6]) in which again the salesman has to fulfill a quota but now nonnegative penalties are associated to the cities and the cost of the salesman tour is the sum of the distance traveled and the penalties for the non visited cities. QTSP corresponds to the case of PCTSP in which all penalties

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⁴ Note that some authors use to name PCTSP the special case that we have called QTSP [5, 8].
are 0. A special case of QTSP is the case in which the amount of merchandise that can be sold in any city is 1; this case coincides with the so called \( k \)-TSP problem, i.e. the problem to find the minimum tour which visits \( k \) cities among the given ones. Moreover this problem is related with the \( k \)-MST problem, that is the problem to find the minimum tree which spans \( k \) nodes in a graph. Clearly, if all weights are equal to 1 and the quota to be achieved is equal to the number of cities, the problem corresponds to the classic TSP.

The QTSP problem and the other problems mentioned above have been thoroughly studied in the past from the point of view of approximation algorithms. In particular, for the \( k \)-TSP problem and for the \( k \)-MST problem a polynomial time algorithm with an approximation ratio 3 has been shown in [15] while, for \( k \)-MST problem, in the non rooted case, an algorithm with ratio 2.5 has been shown in [1]. For the general PCTSP problem a polynomial time algorithm with polylogarithmic performance guarantee has been given in [4, 5].

In this paper we wish to address the on-line version of the QTSP problem, named OL-QTSP. On line versions of other routing problems such as the traveling salesman problem [3], the traveling repairman problem [13, 16, 18], variants of the dial-a-ride problem [2, 13], have been studied in the literature in the recent years. The most updated results regarding these problems can be found in [17]. In the on-line version of QTSP we imagine that requests are given over time in a metric space and a server (the traveling salesman) has to decide which requests to serve and in what order to serve them, without yet knowing the whole sequence of requests, with the aim of fulfilling the quota by traveling the minimum possible amount of time. As it is common in the evaluation of on-line algorithms [14], the performance of the algorithm (cost of serving the requests needed to fulfill the quota), is matched against the performance of an optimum off-line server, that is a traveling salesman that knows all the requests ahead of time and decides which requests to serve and in what order, in order to fulfill the assigned quota. Clearly the off-line server cannot serve a request before its release time. The ratio between the former and the latter is called \textit{competitive ratio} of the on-line algorithm (see [10]).

In the rest of the paper we first provide a formal definition of the problem and we introduce the corresponding notation. Subsequently, in Section 2 we provide the proof that no algorithm for the OL-QTSP problem can achieve a competitive ratio smaller than 2 in a general metric space, and we also give a simple 2-competitive (hence optimal) on-line algorithm. In Section 3 we discuss the case in which the metric space is the halfline. In such case we introduce an algorithm which achieves a competitive ratio 3/2 by not moving at full speed. We also show a matching lower bound. In Section 4 we apply the same ‘slow is good’ paradigm used in Section 3, to the design of an on-line algorithm for the OL-TSP and in such way we derive the best on-line polynomial algorithm known so far, whose competitive ratio is 2.78. In the Conclusion section we illustrate some possible developments and open problems.