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A Unified View in Planning Broadcasting Networks

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1 Introduction

Analog terrestrial broadcasting is a worldwide fully developed technology and operating networks have been deployed in the early years of television. The recent introduction of the Terrestrial Digital Video Broadcasting (DVB-T) is producing a complete restructuring of audio/video networks. The worldwide implementation status of DVB-T can be found in [7]. DVB-T is considered the biggest novelty in television history since it allows a better exploitation of the frequency spectrum (transmitting up to five programs on the same physical channel), supports the High Definition TV and can be integrated with other media (radio, Internet, mobile telephony). However, the development of DVB-T networks is a challenging task for broadcasters, since most part of the (limited) frequency spectrum is filled up by analog networks. Moreover, prescriptive restrictions, such as those regulating frequency assignment at country boundaries (see, e.g., the recent Geneva agreement [14]) impose further limitations to the spectrum utilization. Italy has the most complex terrestrial broadcasting system in Europe, with a percentage of terrestrial-only households close to 85 %. Currently, 3 public national networks, 7 commercial national networks and more than 700 local networks operate with about 23,000 transmitters (in France and Germany they are about 12,000 and 10,000 respectively). In such a congested situation, drawing frequencies from analog networks is mandatory in order to set up digital ones. This is a complicate process, involving three kinds of players: broadcasters, customers and regulation Authorities, each with its own objectives. Broadcasters are interested in minimizing costs and audience loss; customers are afraid of uncomfortable set-up of digital equipments; and the regulation Authorities have to guarantee market fairness (for instance, fixing dates for mandatory analog switch-off, establishing restrictions for major broadcasters, promoting incentives for the customers). The challenge for OR is to provide simulation and optimization methods able to support this process from all such different perspectives.

We look at general broadcasting networks, in which a set of transmitters distributes services to a set of receivers. The transmitters configuration is defined by a number of parameters, such as activation status, emission power in each direction (antenna diagram) and transmission frequency. The receivers configuration depends on service status, antenna orientation and serving transmitter. The general network planning problem consists in establishing suitable configurations for the parameters so as to maximize the customers coverage. Besides the general problem, several subproblems, in which some of the parameters are fixed, are practically relevant for the different market players.

In this paper, we introduce the complete hierarchy of problems which turned out to be relevant in several contextual experiences carried out by the authors related to the migration from analog to digital television in Italy. We also illustrate a generalization of known models and algorithms, elaborated for specific subproblems, so as to provide a unifying view of IP methods for audio-video broadcasting network planning.

The first step towards the application of IP techniques is expressing the receiver coverage condition as a linear function of the received emission powers. In particular, a linear inequality is used to check whether the ratio between overall wanted and interfering signal on a testpoint exceeds a given threshold. This yields a Mixed Integer Linear Programming (MILP) model, known as *Signal-to-Interference* (SIR) model. Unfortunately, the SIR model is weakly structured and may happen to be computationally intractable for state-of-the-art MILP solvers when applied to large scale practical problems [22]. This also agrees with similar experiences reported in closely related contexts, such as GSM [23] and UMTS [3, 12]. However, the SIR model can be considered a starting point to extract some exploitable structure and develop more advanced integer programming techniques. We survey both heuristics and IP reformulations: the former has been successfully applied in practice, while the second represent the current most promising research direction. This concerns the application of standard reformulation techniques to the SIR model: the Dantzig-Wolfe decomposition, which yields a *Set Packing* (SP) model with exponentially many variables [19]; a Benders' like decomposition, which yields a *Set Covering* (SC) model with exponentially many constraints [20].

The paper is organized as follows. In §2 the problem is described; in §3 the hierarchy of problems is presented; in §4 the possible modelling approaches are discussed; section §5 is devoted to heuristic algorithms, while IP reformulations are presented in §6; finally, in §7 some conclusions are drawn.

2 The problem

A *wireless network* distributes its *services* from a set T of *transmitters* over a portion of territory, referred to as *target area*. A transmitter radiates electromagnetic waves in all directions according to its configuration. The basic set of configuration parameters includes geographical location and antenna tower height (*physical* parameters), transmission frequency, emission power, polarization and antenna diagram (*radio-electrical* parameters). The latter represents the relative power transmitted from the antenna in a given set of directions. When the set of directions lies on the plain perpendicular to the antenna tower axis the corresponding radiation plot is referred to as *horizontal* antenna diagram (see Figure 1).

In this paper, the physical parameters are considered known *a priori* and fixed. In fact, a transmitter needs a complex and costly supporting infrastructure; as a consequence, broadcasters establish in advance a set of (relatively few) eligible sites. On the contrary, among the radio electrical parameters, we consider frequencies and powers (emitted in a given set of directions) as decision variables.

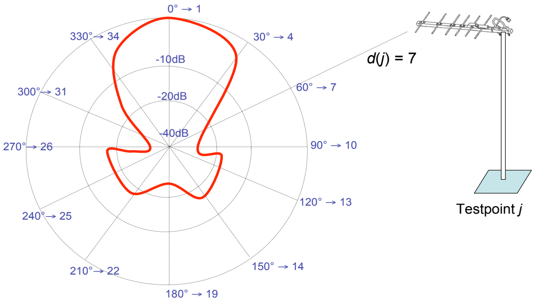

Figure 1: Antenna diagram

The frequency spectrum is subdivided into a set $\mathcal{F} = \{1, \dots, |\mathcal{F}|\}$ of equally sized intervals called *channels* (or *frequencies*). The set of feasible frequencies for transmitter $i \in T$ is denoted by $\mathcal{F}_i \subseteq \mathcal{F}$. It may happen that $\mathcal{F}_i \subset \mathcal{F}$ because of technical or commercial constraints (international agreements at boundaries, licensing, etc.). The *transmission frequency* assigned to transmitter $i \in T$ is denoted by f_i.

For each transmitter $i \in T$, we consider 36 different power values corresponding to the subdivision of the horizontal antenna diagram into equally angled sectors. Thus, P_i^d denotes the emission power of transmitter i along direction $d \in \{1, \dots, 36\}$. Each P_i^d ranges in the interval $[P_i^{\min}, P_i^{\max}]$ (*power bounds*). In order to give a feasible antenna diagram, the 36 emission powers must satisfy three technological constraints: *adjacency* constraints, *front/rear* constraints and *symmetry* constrains. Adjacency constraints state that the power ratio between adjacent sectors must not exceed a given threshold $\Delta \geq 1$:

$$\frac{1}{\Delta} \cdot P_i^{d-1} \leq P_i^d \leq \Delta \cdot P_i^{d-1} \qquad d \in \{1, \dots, 36\} \tag{1}$$

where $d-1$ is taken modulo 36.
Front/rear constraints ensure that the power ratio between opposite sectors must not exceed the front/rear threshold Γ:

$$P_i^d - \Gamma \cdot P_i^{d+18} \leq 0 \qquad d \in \{1, \dots, 36\} \tag{2}$$

where $d+18$ is evaluated modulo 36.

Finally, symmetry constraints imply that the power emitted in direction d must be equal to the power emitted in direction $(d+h)$ modulo 36, for some pair d, h. For each $i \in T$, the set of technological constraints define a polyhedron $\mathcal{D}_i \subset \mathbb{R}^{36}$.

A power vector $\boldsymbol{P}_i = (P_i^1, \dots, P_i^{36}) \in \mathcal{D}_i$ and satisfying the power bounds is a *feasible antenna diagram*. An antenna diagram is said to be *fixed* if constraints (1) and (2) are in form of equality. A transmitter is *switched off* if $\boldsymbol{P}_i = (0, \dots, 0)$.

The target area is decomposed into a set Z of "small" areas called *testpoints* (TPs). Each testpoint, identified by its coordinates, represents the behavior of a receiver located anywhere inside it. A revenue u_j is defined for each TP j, typically related to the number of customers in j (for $S \subseteq Z$, $u(S) = \sum_{j \in S} u_j$).

The signal emitted by a transmitter i at a given power arrives in a TP j attenuated by a factor depending on the distance between i and j and terrain orography. In detail, the power density received in TP j from transmitter i is expressed by $a_{ij} \cdot P_i^{d(j)}$ $(watt/m^2)$, where $d(j) \in \{1, \dots, 36\}$ is the antenna diagram sector including j (Figure 1) and $a_{ij} \in [0,1]$ is defined through a propagation model (see [28]). The matrix $[\boldsymbol{A}] = [a_{ij}]_{i \in T, j \in Z}$ is known as the *fading matrix*.

The set of signals received in a TP j is denoted by $T(j) = \{i \in T : a_{ij} \neq 0\} \subseteq T$. Figure 2 represents a typical situation in a testpoint. The x-axis displays the arrival time of each signal in the testpoint, which is proportional to the distance of the emitting transmitter. Different shades denote signals at different frequencies. Among all arriving signals exactly one is selected as reference signal (*reference transmitter*) by the receiver. Consequently, the receiver will be tuned on the reference transmitter emission frequency. The reference transmitter is the major (possibly the unique) candidate to distribute the service in TP j.

A TP j is said to be *covered* if the service is received "clearly". The coverage evaluation depends on system type (i.e., analog or digital; audio or video) and receiver behavior. However, a general model applies to most practical settings. In practice, the receiver discriminates between signals arriving in j dividing them into three sets: *useful*, *interfering* and *indifferent* signals. If the reference signal h has frequency f, the set of useful signals contains all signals in $T(j)$ that can be constructively combined at frequency f. In DVB-T, thanks to the *Orthogonal Frequency Division Multiplexing*, this set may contain several signals (see [11] for details), while, in analog broadcasting, this set contain exactly one signal (typically, the reference signal h). The set of useful signals is denoted by $W(j, h, f)$.