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Set Theory: Recent Trends and Applications

edited by Alessandro Andretta

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Contents

Mathematics subject classification

Disjoint Non-Free Subgroups of Abelian Groups: **20K20**, 20K27, 03E05, 03E75.

Global Complexity Results: **03E05**, 06A07, 03E35.

Absoluteness for Universally Baire Sets and . . . : **03E47**, 03E40, 03E55.

Large Cardinals and L-like Universes: **03E45**, 03E55.

Applications of Regular Filters and . . . : **03C55**, 03E05.

Set Theory of Infinite Imperfect Information Games: **03E60**, 03E15, 91A05.

Forcing with F_σ- and with Summable Filters: **03E05**, 03E35, 06E05, 40A05.

Reasonably Complete Forcing Notions: **03E40**, 03E35, 03E17.

Preface

Set theory started with a paper of Georg Cantor published on Crelle's Journal in 1874, and was developed in the following decades by Cantor himself and many other mathematicians, including Dedekind, Gödel, von Neuman, and Zermelo. By the middle of the twentieth century, set theory had become the common language in which all mathematics is formulated, but it was only after Paul Cohen's work on the Continuum Hypothesis in 1963 that the subject reached full maturity. After Cohen's seminal work, a veritable explosion of results followed, and now, some forty years after, there is no sign of slowing down, with a seeming inexhaustible slew of new techniques and ideas, and unexpected applications to other areas of mathematics.

Due to the extreme richness of the subject, it is simply not possible to give a balanced overview of the feverish set theoretic activity in a reasonably sized collection of papers. The present volume, a collection of papers written by some of the foremost set theorists, presents a snapshot of some of the current exciting trends and developments in the subject, making it valuable both for the beginning graduate student as well as for the experienced researcher.

All of the papers have been refereed according to the usual standards for international math journals, and I would like to thank all of the Authors for their contributions and all of the Referees for their excellent job.

Alessandro Andretta

Disjoint Non-Free Subgroups of Abelian Groups

Andreas Blass and Saharon Shelah

Contents

1. Introduction

In a discussion between the first author and John Irwin, the question arose whether every non-free, separable, torsion-free abelian group has two disjoint non-free subgroups. (Of course, in the context of subgroups, "disjoint" means that the intersection is $\{0\}$.) The main result of this paper is a strong affirmative answer. To state the result in appropriate generality, we need some terminology.

Convention 1.1. *All groups in this paper are understood to be abelian and torsion-free. In particular, "free group" means "free abelian group".*

Definition 1.1. *The* non-free rank *of a group G, written* $\mathrm{nfrk}(G)$, *is the smallest cardinal κ such that G can be split as the direct sum of a group of rank $\leq \kappa$ and a free group.*

Theorem 1.1. *If* $\mathrm{nfrk}(G)$ *is uncountable, then G has* $\mathrm{nfrk}(G)$ *pairwise disjoint, non-free subgroups.*

Recall that any countable, separable group is free. It follows that, if G is separable and not free, then $\mathrm{nfrk}(G)$ is necessarily uncountable, so the theorem applies to G. It gives not only two disjoint non-free subgroups as in the original question but $\mathrm{nfrk}(G) \geq \aleph_1$ of them.

The number of disjoint, non-free subgroups obtained in the theorem is the most one could hope for. Indeed, if $G \cong H \oplus F$ where F is free and H has rank and therefore cardinality equal to the infinite cardinal $\mathrm{nfrk}(G)$, then any non-free subgroup S of G must have a non-zero intersection with H. Otherwise the projection to F would map S one-to-one into the free group F and it would follow that S is free. Therefore, disjoint non-free subgroups of G must intersect $H - \{0\}$ in disjoint non-empty sets. So there cannot be more such subgroups than $|H| = \mathrm{nfrk}(G)$.

Although the theorem gives an optimal result for separable groups, the fact that it does not explicitly mention separability raises another question:

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