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A derivative-free algorithm for nonlinear programming

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Abstract

In this paper we consider nonlinear constrained optimization problems in case where the first order derivatives of the objective function and the constraints can not be used. Up to date only a few approaches have been proposed for tackling such a class of problems. In this work we propose a new algorithm. The starting point of the proposed approach is the possibility to transform the original constrained problem into an unconstrained or linearly constrained minimization of a nonsmooth exact penalty function; this approach shows two main difficulties: the first one is the nonsmoothness of this class of exact penalty functions; the second one is the fact that the equivalence between stationary points of the constrained problem and those of the exact penalty function can be stated only when the penalty parameter is smaller than a threshold value which is not known *a priori*. In this paper we propose a derivative-free algorithm which overcomes the preceding difficulties and produces a sequence of points that admits a subsequence converging towards Karush-Kuhn-Tucker points of the constrained problem. In particular the proposed algorithm includes an updating rule for the penalty parameter which, after, at most, a finite number of updatings, is able to determine a "right value" of the penalty parameter. Numerical results on a set of test problems are reported which show the viability of the proposed algorithm.

AMS subject classification. 65K05, 90C30, 90C56

Keywords: Derivative-free optimization, constrained optimization, nonlinear programming, non-differentiable exact penalty functions.

1 Introduction

We consider the following problem

$$\begin{array}{ll} \min & f(x), \\ s.t. & g(x) \le 0 \\ & Ax \le b, \end{array} \tag{1}$$

where $x \in \Re^n$, $f : R^n \to R$, $g : R^n \to R^m$, $A \in R^{p\times n}$, $b \in R^p$ and we assume that f and g are twice continuously differentiable on R^n. We denote by $a_j^\top$, $j = 1, \dots, p$, the rows of matrix A and by

$$\mathcal{F} = \{x \in \Re^n : \ Ax \le b, \ g(x) \le 0\}$$

the feasible set of Problem (1). We assume that the derivatives of the objective and nonlinear constraint functions cannot be neither calculated nor approximated explicitly. Indeed, in many engineering problems the analytic expressions of the functions defining the objective and constraints of the problem are not available and their values are computed by means of complex simulation computer programs. Though this does not necessarily means that derivatives cannot be computed, this is the case when the source code of the simulation routines is unavailable. For further motivations on the necessity of using derivative-free methods we refer the reader to the survey paper [13].

In the literature, some globally convergent derivative-free methods for the solution of Problem (1) have been proposed. In [18] a pattern search algorithm is used within a sequential augmented Lagrangian approach. Essentially, the method embeds the pattern search algorithm proposed in [17] within the augmented Lagrangian method [5] which is the basis for the subroutine AUGLG in the LANCELOT optimization package.

In [1] the filter method proposed in [9] is adapted to include a patter search minimization strategy. Basically, the method employs a "filter" for acceptance of the points produced by the patter search local optimizer.

In [2] a so-called *extreme barrier* approach, namely the constrained problem is converted to an unconstrained one by setting the objective to infinity for infeasible point, is employed. To minimize

this extreme barrier function, the authors propose an extension of the generalized pattern search class of algorithms which allows local exploration in an asymptotically dense set of directions.
In this paper we propose an algorithm which stems from an idea different from the preceding ones. In a way similar to [18] and [2], we obtain an unconstrained reformulation of Problem (1) by means of a suitable merit function. Then, the merit function has been minimized by employing a line-search derivative-free approach.
In particular, the method that we propose hinges on the following two main points:

- the use of non-differentiable exact penalty functions (see [8] for further details) to obtain an unconstrained reformulation of Problem (1);

- the employment of the recently proposed algorithm [14] for linearly constrained finite minimax problems.

In spite of the apparent simplicity, the above two approaches cannot be straightforwardly combined to produce an efficient and globally convergent derivative-free algorithm. Indeed, there are both theoretical and computational aspects which should be carefully taken into account when combining the two approaches. From a theoretical point of view, it is important to note that an exact penalty function enjoys its exactness properties only if the penalty parameter is below a certain threshold value which is not known "a priori". For this reason, an algorithm using an exact penalty function must include a suitable updating rule for the penalty parameter ϵ which, after a finite number of reductions, be able to spot a right value for ϵ so as to convey the desirable exactness properties to the penalty function being minimized. Apart from this appreciable theoretical aspect, from a computational point of view merit functions can suffer from ill-conditioning. What is more, since derivative-free codes rely only on the evaluations of the objective function to carry on the minimization process, they are more easily disturbed by the negative effects of ill-conditioning. Therefore, adequate precautions should be taken into account in order to limit as much as possible this undesirable effect. In particular, in an effort to make the penalty function more well-behaved, the following two points have been thoroughly investigated:

- the structure of Problem (1), by explicitly handling the linear inequality constraints;
- the structure of the penalty function itself, by introducing a different kind of barrier term.

The paper is organized as follows. In Section 2, the exact penalty function approach is introduced and discussed. Section 3 is devoted to the description of a smooth approximation technique along with some preliminary properties. In Section 4, the derivative-free method is reported and its global convergence is studied. Finally, Section 5 reports some preliminary numerical experience with the proposed derivative-free method.
We conclude this introductory section, by noting that, as concerns Problem (1), the presence of the linear inequality constraints allows us to define necessary optimality conditions under somewhat weaker assumptions than usual. In particular, under a weakening of the Mangasarian-Fromowitz constraint qualification condition, any local solution $\bar{x}$ of Problem (1) satisfies the following (KKT) necessary conditions.

Proposition 1 *Let $\bar{x} \in \mathcal{F}$ be a local solution of Problem (1) and assume that a vector $z \in \{d \in \Re^n : a_j^\top d \le 0,$ for all j s.t. $a_j^\top d = b_j\}$ exists such that*

$$\nabla g_i(\bar{x})^\top z < 0, \qquad \textit{for all } i \textit{ s.t. } g_i(\bar{x}) = 0.$$

Then,

$$\begin{aligned} &\nabla f(\bar{x}) + \nabla g(\bar{x})\bar{\lambda} + A^\top \bar{\mu} = 0 \\ &\bar{\lambda}^\top g(\bar{x}) = 0, \quad \bar{\lambda} \ge 0, \\ &\bar{\mu}^\top (Ax - b) = 0, \quad \bar{\mu} \ge 0, \end{aligned} \tag{2}$$

for some vectors $\bar{\lambda} \in R^m$ and $\bar{\mu} \in \Re^p$.

Proof. The proof follows by considering Propositions 3.3.11 and 3.3.12 in [3] along with the Motzkin theorem of the alternative [19]. □